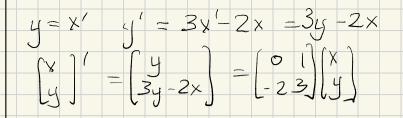
Pre-class Warm-up!!!

Which of the following systems of equations is equivalent to the 2nd order equation x'' - 3x' + 2x = 0?

a. $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ b. $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$ c. $\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

d. None of the above



We have the characteristic polynomial $r^2 - 3r + 2$ (of the d. e.) and the characteristic polynomial of [0] det[-23] $det[-23-\lambda]$ $= -\lambda(3-\lambda) + 2 = \lambda^2 - 3\lambda + 2$. They are the come!

Section 7.2 Matrices and linear systems We learn about: • writing a linear system of equations in vector form • several theorems similar to ones for higher order d.e.'s we have already seen • the Wronskian again. $instead of x_i' = P_{11} \times P_{12} \times P_{2} + \cdots + P_{12}$

Xn

A system of equations X' = PX + F is homogeneous if $F = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0 = 0$

 IFX_1, X_2 Taking derivatives is a linear operator: are vector valued functions, C1, C2 are scalars then

The principle of superposition of solutions:

If X_1, X_2 are collitions to a homogeneous system then so is $c_1 X_1 + c_2 X_2$

/Theorem 1 The solutions to a homogeneous system form a vector space.

Theorem 3 The space has dimension n if P and F are continuous.

The Wronskian of vector valued functions X_1, \ldots, X_n is

 $det [X_1 - X_n] = W(t)$

Theorem 2 (a) If X_1, ..., X_n are dependent then W = 0. (b) If they are also solutions of a homogeneous linear system and they are independent, then W is never 0.

Proof (a) IF they are dependent then there is a nonzero dependence relation between the columns of $[X_1] - [X_n]$ (one is a linear condination of some other) This means the det. is O. The connection with the Wronskian of scalarvalued functions f_1, ..., f_n.

Fn

That Wronskian was det F.

In converting from a high order d.e. in one variable we introduced variables $x_1 = x_1$, $x_2 = x_1'$, $x_3 = x_2'$

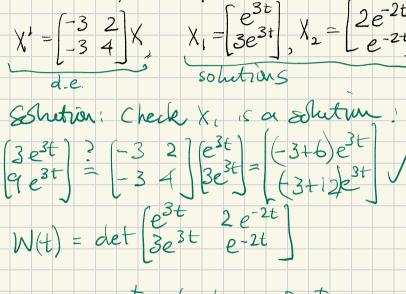
- F(n-)

photocing vectors (X1) X x2 - X1 Xn) (Xn-D)

The two Wronskians are the same when we convert a high order d.e. in one variable to a first order system. in several variables.

Page 384 question 14.

Verify that the given vectors are solutions of the differential equation. Use the Wronskian to show that they are independent.



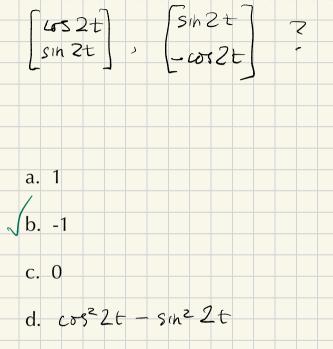
= et - Get = -5 et This is not the zero function. In fact it is never D,

Thus XI, X, are independent.

Page 384 question 23. Find a particular solution of the system in question 14 that satisfies $x_1(0) = 0$, $x_2(0) = 5$ component 1 of the solution coust 2 Solution; We look for a solution $\begin{array}{c|c} A & z \\ 3e^{3t} & + B & e^{-2t} \\ \hline \end{array} \\ \hline \end{array} \\ \begin{array}{c} Puf \\ t = O \\ \hline \end{array}$ $A\begin{bmatrix}1\\3\end{bmatrix}+B\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}0\\5\\3\end{bmatrix}\begin{bmatrix}1&2\\3\end{bmatrix}A\end{bmatrix}=\begin{bmatrix}0\\5\end{bmatrix}$ The particular solution is 2e3t - 2e-2t best - e-2t

Question.

What is the Wronskian of the functions



e. None of the above.

- For each B = [b] there is a Theorem 1 of section 7.1. In the first order linear system X' = PX + F if the functions P and F are continuous then, given unique linear combination numbers a, b_1, ..., b_n, there is a unique solution satisfying $C_1 X_1(a) + \dots + C_4 X_d(a) = 3$ $x_1(a) = b_1, x_2(a) = b_2, \dots, x_n(a) = b_n.$ $\left(\begin{array}{c} X_{1} \\ \end{array}\right) \\ - \left(\begin{array}{c} X_{d} \\ \end{array}\right) \\ \left(\begin{array}{c} q \\$ We conclude: (C_1, \ldots, c_d) . Theorem 3. It follows that (X, [...-]X& (is The space of solutions of a homogeneous first order linear system in n variables has dimension n. square, so d=n, Deduction of this: Take a bairs X1,..., Xd for the space of solutions.